



A New Look at the Phase-Locked Oscillator*

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Summary—The uses of phase-locked oscillators are briefly reviewed. A simple automatic-phase-control (APC) system is analyzed as a servomechanism analog. Three major characteristics of the system are considered: the lock range, the capture range, and the filter bandwidth. The lock range is the total drift in the unlocked oscillator frequency which can be exactly compensated by the locked system. The capture range is the largest unlocked frequency difference at which synchronization, or lock-in, will occur. The filter bandwidth of the system expresses the performance of the system as a low-pass filter with respect to FM noise components existing in the input to the system and as a high-pass filter with respect to FM noise components generated within the output oscillator.

The mutual interdependence of these characteristics and the various quantities affecting each one are discussed. The conditions for stable operation of the system are established. The unlocked, locking-in, and locked conditions of operation and the effects of the low-pass filter are discussed. Simple design criteria are established.

INTRODUCTION

THE phase-locked oscillator system has been analyzed many times in the past.¹⁻⁷ Most of these analyses have been in classical mathematical

terms. In this paper the author is attempting a new approach to the problem, utilizing concepts which are well understood by people familiar with servomechanism and feedback system design. It is hoped that this approach will produce a clearer understanding of the system, both qualitative and quantitative.

USES OF PHASE-LOCKED OSCILLATORS

There are many varied uses for a phase-locked oscillator or automatic-phase-control (APC) system as it is sometimes called. An APC system can be used in a receiver to increase the power level and attenuate the noise of a weak FM signal. A similar system, involving somewhat different design parameters, can be used to reduce the jitter or frequency noise of a high-powered oscillator. In the field of frequency measurement and synthesis, in particular, the APC system is extremely useful.

Many frequency measurement and synthesis systems involve the generation and selection of a single frequency signal. Because of the generation process, a poor signal-to-noise ratio (SNR) results—the noise in this case taking the form of adjacent frequency components (or sidebands) and small deviation FM or phase jitter of the desired component. A phase-locked oscillator can serve as a filter of arbitrarily narrow bandwidth (arbitrarily high Q) for the selection of the desired signal, attenuating the unwanted components and reducing the phase jitter.

DESCRIPTION OF A PHASE-LOCKED OSCILLATOR SYSTEM

Fig. 1 shows the basic components of an elementary APC system. There are many ramifications of this basic system, some using frequency dividers or multipliers,

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¹ R. Ley, "Phase synchronization of an oscillator, application to a continuously variable oscillator of high stability," *Ann. Radioelec. (Paris)*, vol. 13, pp. 212-233; July, 1958.

² T. J. Rey, "Effects of the Filter in Oscillator Synchronization," Lincoln Lab., M.I.T., Lexington, Mass., Tech. Rep. No. 181 (ASTIA 133854); May, 1958.

³ R. Leek, "Phase-lock A.F.C. loop, tracking signals of changing frequency," *Electronic and Radio Engr.*, vol. 34, pp. 114 and 177; April and May, 1957.

⁴ D. Richman, "Color-carrier reference phase synchronization accuracy in NTSC color television," *Proc. IRE*, vol. 42, pp. 106 and 288; January, 1954.

⁵ W. J. Gruen, "Theory of A.F.C. synchronization," *Proc. IRE*, vol. 41, pp. 1043-1049; August, 1953.

⁶ G. W. Preston and J. C. Tellier, "The lock-in performance of an A.F.C. circuit," *Proc. IRE*, vol. 41, pp. 249-251; February, 1953.

⁷ E. Labin, "Theory of synchronization by control of phase," *Phillips Res. Rep.* vol. 4, pp. 291-315; August, 1949.

and some using offset comparison frequencies; but we shall restrict our attention to the simple system shown here. No great generality will be lost by this procedure. The system contains: an oscillator with a nominal frequency equal to the desired output frequency; some form of reactance modulator or other means for voltage control of the oscillator frequency; a phase detector which compares the outputs of the oscillator and the reference source; and a low-pass filter which filters the output voltage of the phase detector before it is applied to the reactance modulator.

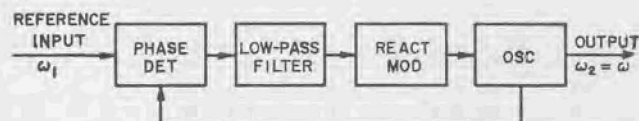


Fig. 1—Block diagram: automatic phase-control system.

The operation of the system can be understood qualitatively by assuming that the oscillator frequency is equal to that of the reference. The phase-detector output is then a dc voltage dependent on the phase difference between the output signals of the oscillator and the reference. This voltage is applied through the low-pass filter to the reactance modulator and thereby governs the oscillator frequency. If the oscillator frequency tends to change, this attempted change is first felt as a phase-difference change in the phase detector. This produces a change in phase-detector output voltage which acts to hold the oscillator frequency constant. As the oscillator drifts, its output phase, relative to that of the reference, will drift, but its average frequency will remain fixed. The system operates exactly like a positional servomechanism wherein, for constant input position, the output position is exactly equal to the input, with zero steady-state error.⁸ To understand the lock-in performance of the system and its quantitative behavior with respect to noise and drift requires a closer look.

SERVO ANALYSIS OF A PHASE-LOCKED OSCILLATOR

A block diagram of the simple phase-locked oscillator system, which lends itself more readily to analysis, is shown in Fig. 2. This block diagram, incidentally, is identical to that for a positional servo. The circled Σ 's represent summation points where various signal variables (voltages or frequencies) are combined. Frequency noise (incidental FM or jitter) contributed by the reference source is represented by N_1 . The $1/S$ term represents the integration of frequency difference to phase difference which occurs in the phase detector. It is this integration which gives the system its unique properties and distinguishes it from the more familiar automatic-

frequency-control (AFC) system. In an AFC system, the frequency of the oscillator is compared to a reference frequency. For example, the resonant frequency of a passive circuit, and the frequency difference—not phase difference—is used to generate a signal which tends to reduce the frequency difference. Such a system requires a small, but finite, error of the controlled variable (the output frequency) in order to operate. The phase-lock system, on the other hand, requires no steady-state error of the controlled variable, but instead utilizes an error in the *integral* of the controlled variable, *i.e.*, an error in phase difference. The gain of the phase detector in volts per radian is represented by K_1 . The frequency characteristic of the low-pass filter is indicated by $F(S)$, and K_2 represents the gain of the reactance-modulator-oscillator combination in radians per second, per volt. N_2 represents noise voltage at the input to the reactance modulator, and Ω includes both the detuning of the oscillator and frequency noise in its unlocked output.

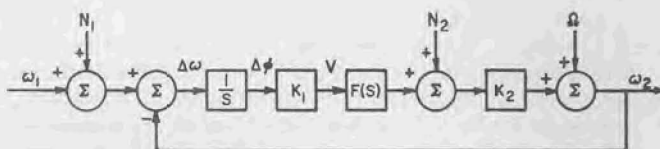


Fig. 2—Servo block diagram.

Lock Range

There are several quantities of interest for this system. Among the most important are the lock range, the capture range, and the filter bandwidth. From a knowledge of these quantities, much of the behavior of the system can be calculated, such as the static phase error, the response to an input transient, etc. The lock range is the total drift in unlocked output frequency which can be exactly compensated by the system. For a multiplier or divider system, the lock range is usually chosen so that all expected drifts in the oscillator frequency due to component changes and ambient temperature variations can be compensated as the phase-detector output traverses its entire range. From Fig. 2 it can be seen that the lock range is equal to the range of the reactance-modulator-oscillator combination caused by the maximum voltage range of the phase detector. For a linear reactance modulator and a linear phase detector operating over a range of π radians, the lock range would equal $\pi K_1 K_2$ radians per second. That is, a phase-difference change of π radians produces a voltage change of πK_1 volts out of the phase detector, which in turn produces a change of $\pi K_1 K_2$ radians per second in the frequency of the reactance-modulator-oscillator combination. For a linear reactance modulator and a typical nonlinear balanced phase detector, however, which has a maximum output swing of $2K_1$ volts, the lock range would be $2K_1 K_2$ radians per second.

A given total lock range can be achieved with a large K_1 and a small K_2 or vice versa. Whichever gain should

* H. Chestnut and R. Mayer, "Servomechanisms and Regulating System Design," John Wiley and Sons, Inc., New York, N. Y.: 1950 and 1955.

be made larger depends on the noise susceptibilities of the various components. Since, in a practical system, the input to the reactance modulator is usually the most noise-sensitive point in the system, a large K_1 and a small K_2 are desirable. This is achieved with a high-output phase detector and a "stiff" reactance-modulator-oscillator combination. Indeed, a "stiff" oscillator offers many advantages in a system for removing phase modulation from the reference signal. An oscillator whose frequency is hard to pull usually has good long and short-term stability and thus requires a smaller lock range and produces a cleaner output than a "loose" oscillator. For these reasons, a quartz crystal oscillator is sometimes used at the output of a phase-locked multiplier.

Capture Range

The capture range is the largest unlocked frequency difference at which the system will lock-in. This capture range cannot be larger than the lock range, but it can be smaller. To gain a clear understanding of the operation of the system, let us ignore for a time the low-pass filter. That is, let us assume that the cutoff frequency, ω_f , of this filter is greater than certain other frequencies of interest. The reason for this will be more apparent later. *For this condition the capture range is equal to the lock range.*

Fig. 3 is a diagram which aids in obtaining a physical picture of the operation of the system—both for the locked and the unlocked conditions. Oscillator frequency is plotted along the vertical axis, and phase difference is plotted along the horizontal axis. The curves represent the frequency of the oscillator as a function of oscillator-reference phase difference and repeat for every integral multiple of 2π . There exists a family of these curves, each one corresponding to a different rough tuning of the oscillator.

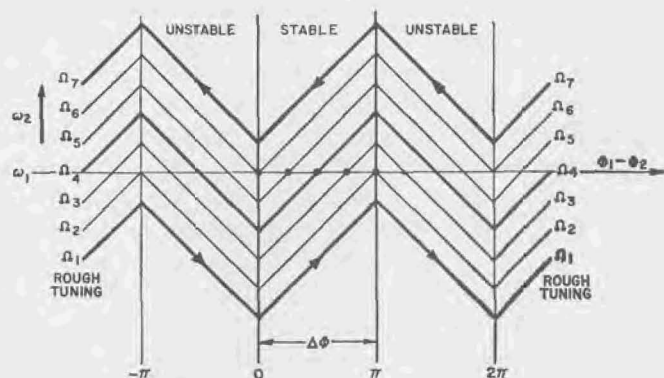


Fig. 3—Composite locking diagram.

In order for locking to occur, the rough tuning must be such that the oscillator frequency curve intersects the ω_1 line (*i.e.*, $\omega_2 = \omega_1$). Assume that Ω_4 in Fig. 3 represents the oscillator rough tuning. There are two possible intersections of the Ω_4 curve with the ω_1 line. Only one

of these intersections is stable, the other intersection corresponding to a condition of positive rather than negative feedback. The stable intersection is indicated in Fig. 3. This intersection may be called the locked operating point and indicates the phase difference required to maintain lock (the static phase error). If the rough tuning is varied (or drifts) through Ω_3 to Ω_2 , the oscillator will remain locked and the operating point will move toward one end of the stable region. If the rough tuning is varied still further, say to Ω_1 , the oscillator will unlock and the operating point will move along the Ω_1 curve in the direction indicated by the arrows, producing continuous frequency modulation of the oscillator.

Now let us reverse the procedure. Assume that the oscillator tuning has been brought to Ω_1 from some far-distant value. Since the Ω_1 curve does not intersect the ω_1 line, locking is impossible and, as mentioned before, the operating point moves along the Ω_1 line. The nature of this motion can be understood qualitatively by the following physical argument. Since the frequencies of the oscillator and reference are not equal, the output of the phase detector is a beat voltage. The "frequency" of this beat will equal the difference between the oscillator and reference frequencies. When the operating point on the Ω_1 curve is in the region furthest removed from ω_1 , the beat frequency is high and the operating point moves rapidly. When the operating point is in the region near ω_1 , the beat frequency is low and the operating point moves slowly. The oscillator frequency "hesitates" for a time as it nears the reference frequency. Then it sweeps rapidly away and back again. Appendix II shows that, under these conditions, the output voltage of the phase detector is a series of back-to-back exponentials, with flat regions as the operating point hesitates near ω_1 and sharp cusps as the operating point moves rapidly down one side of the phase-detector characteristic and up the other. As the tuning is changed from Ω_1 toward Ω_2 , the hesitation lasts longer and longer until, at Ω_2 , the operating point stops, right at the edge of the lock range, and the oscillator is locked.

For the case considered, *i.e.*, neglecting the low-pass filter, it can be shown that the capture range is equal to the lock range.⁷ That is, if the oscillator frequency vs phase-difference line intersects the desired frequency, the oscillator will lock at the stable point of intersection.

Filter Bandwidth

The filter bandwidth of the system represents its behavior as a "frequency noise" or "jitter" filter. This can be best expressed in terms of a cutoff frequency ω_c . It can be seen in Fig. 2 that the system behaves as a low-pass filter with respect to input noise N_1 . That is, components of N_1 with rates below ω_c , *i.e.*, slowly varying changes in the reference frequency, appear directly in the output, whereas components with rates above ω_c are attenuated. Conversely, the system behaves as a high-pass filter with respect to internal noise N_2 . That

is, components of N_2 with rates below ω_c are attenuated in the output, whereas components with rates above ω_c appear directly. This topic is discussed further in Appendix I.

In a narrow-band system, the output oscillator does not follow rapid excursions of the reference frequency such as rapid FM or jitter, but any internally generated rapid excursions appear directly. In a wide-band system, on the other hand, the output oscillator follows rapid excursions of the reference frequency, but internally generated rapid excursions are attenuated. Depending on the purposes of the system, whichever noise source is most objectionable determines whether ω_c should be made small (narrow band) or large (wide band). For example, if the purpose of the system is to select the long-term average of the input frequency and remove incidental FM or jitter existing in the reference, an inherently clean oscillator (such as a crystal oscillator) should be chosen and ω_c should be made as small as possible. On the other hand, if the purpose of the system is to reproduce the frequency excursions of the input³ or to remove incidental FM in the output oscillator (such as a klystron oscillator), ω_c should be made as high as possible. However, one is not free to adjust the bandwidth without, at the same time, affecting the lock range or the capture range.

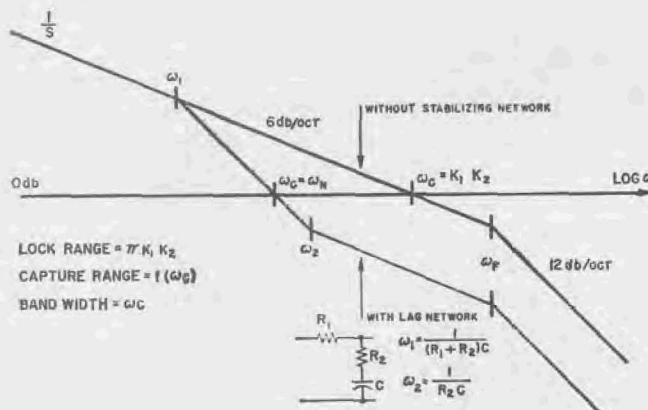


Fig. 4—Open loop gain.

Fig. 4 shows a plot of the open-loop gain of the system in decibels vs logarithmic frequency. Because of the $1/S$ integration term, the loop gain decreases with frequency at a rate of 6 db per octave, intersecting the zero-decibel (unity gain) line at a frequency $\omega_c = K_1 K_2$. This is the cutoff frequency of the closed-loop system. That is, the transfer characteristic of the system equals unity ($\omega_2 = \omega_1$) for input variations with rates below ω_c and falls off for rates above ω_c . The low-pass filter is assumed to be a simple RC section with a cutoff frequency of ω_f (shown greater than ω_c). This filter causes an increase in the rate of open-loop and closed-loop attenuation to 12 db per octave above ω_f .

This open-loop gain-frequency diagram completely defines the characteristics of the system. The lock range

is equal to $\pi K_1 K_2$, the capture range is equal to the lock range, and the cutoff frequency ω_c is equal to $K_1 K_2$. The static phase-error constant is $1/K_1 K_2$ radians phase difference for each radian per second attempted frequency difference. Lock range, capture range, and bandwidth all increase in proportion to $K_1 K_2$. Fortunately, this is often advantageous. A system for locking a klystron oscillator, for example, should have a large $K_1 K_2$ to accommodate wide drift in the klystron frequency. A large $K_1 K_2$ product automatically produces a desirable wide capture range and the wide bandwidth necessary for noise attenuation. Similarly, designing a system with small $K_1 K_2$ to accommodate a crystal oscillator automatically produces a small capture range and a narrow bandwidth.

Effect of the Low-Pass Filter

Although the system bandwidth cannot be made greater than $1/\pi$ times the lock range, one might think that the bandwidth could be reduced by making the cutoff frequency, ω_f , of the low-pass filter lower than $K_1 K_2$ but one has to be a little careful about this. Several changes occur as the filter cutoff frequency, ω_f , is lowered. The phase shift at the frequency at which the loop gain is unity gets closer to $\pi(180^\circ)$, and the system transient response to a step input develops underdamped ringing.⁹ Another important happening is a reduction of the capture range. The unlocked oscillator frequency must be brought nearer to the reference frequency than indicated by the lock range, for locking to occur. A hysteresis effect exists. For this case it can be shown that the reduction in capture range is approximately equal to the reduction in system bandwidth (unity-gain frequency) caused by a lowering of ω_f .^{1,2}

A better method of narrowing the bandwidth involves the use of a lag (integral-compensation) network as the low-pass filter.⁸ The effect of this network is shown in the dashed curve of Fig. 4. Such a network also reduces the capture range, but not as drastically as the simple RC filter discussed above. It can be shown that the use of a lag network with constants chosen to insure a minimum noise bandwidth (for the chosen system bandwidth) will produce a reduction in capture range approximately equal to the square root of the reduction in system bandwidth.^{2,5} (See Appendix I.)

A reduction in capture range can possibly be tolerated if the operator of the system is required to lock the oscillator before use or if a nonlinear lag network¹ is used. However, such a system may be only conditionally stable. If the oscillator drifts outside the capture range—still remaining within the lock range—a small perturbation may cause it to unlock. The system is also conditionally stable in the sense that any nonlinearity or shift in K_1 or K_2 can cause the open-loop unity-gain frequency to move to an unstable region.

It is usually desirable to have the capture range as

⁹ See Appendix I.

nearly equal to the lock range as possible, to insure locking whenever the oscillator tuning is within the lock range. This requires a low-pass filter with a cutoff frequency greater than K_1K_2 .

Summary

To summarize the foregoing results: The lock range of the system in radians per second is equal to πK_1K_2 , π times the product of the gains of the phase detector and reactance-modulator-oscillator combination. The bandwidth of the system is determined by the cutoff frequency, ω_c , the frequency at which the open-loop gain falls to unity. This is equal to K_1K_2 in the absence of a low-pass filter. The capture range of the system is a function of the cutoff frequency, ω_c . With no low-pass filter, the capture range is equal to the lock range. With a simple RC filter reducing ω_c , the capture range is reduced by approximately the same factor. With an optimized lag network reducing ω_c , the capture range is reduced by approximately the square root of the bandwidth reduction.

DESIGN PROCEDURE

The usual procedure is to design the best oscillator economically feasible to produce the desired output frequency. The reactance modulator may be a reactance tube, a voltage-variable inductor or capacitor, or the like. The phase detector may be a balanced type to reduce amplitude sensitivity or a simple tuned-circuit-diode rectifier if the amplitudes involved are constant. Use of a limiter is desirable since amplitude variations in the phase detector can result in frequency modulation of the output oscillator. The product K_1K_2 , preferably with large K_1 and small K_2 , should be chosen to insure locking for all expected drifts in oscillator or reference frequency. For a maximum capture range, the low-pass filter should have a cutoff frequency slightly greater than K_1K_2 . If a wider bandwidth is desired to attenuate internal noise, the lock range may be made greater than necessary. This will also reduce the phase error required to correct a given frequency error.

If a small static phase error and a narrow bandwidth are desired and a reduction in capture range can be tolerated, a lag network should be used.⁹

It should be emphasized that in actual practice the parameters, K_1 and K_2 , may be variable. The common balanced phase detector, for example, has a sensitivity (K_1) which depends on the phase-difference operating point. Similarly, most reactance-tube controlled oscillators have sensitivities (K_2) which depend on the reactance-tube bias, the *O*-bias (high gm) region being the more sensitive. Therefore, the bandwidth and stability may vary with the lock operating point. All possible values of K_1 and K_2 should be considered in the design.

Ordinarily K_1 , K_2 , and the low-pass filter characteristic are chosen for the best compromise with respect to lock range, capture range, static phase error, transient response to input signal variations, input noise rejection,

internal noise rejection, and economy, for the particular application. Often a great deal of subjective experimenting is required. The criteria mentioned above should serve as a useful guide, however.

CONCLUSION

In conclusion, the phase-locked oscillator is a versatile device. It can be used to increase the power level and SNR of a weak, jittery signal or to reduce short-term frequency excursions of either the reference source or the oscillator itself. The techniques described have been used successfully to multiply standard frequencies from 100 kc to 1 mc, 5 mc, 10 mc, and 100 mc with locked crystal oscillators, and to 1000 mc with a locked klystron oscillator.^{10,11}

APPENDIX I

Analysis of the Low-Pass Filter Effect

System Transfer Function: In Fig. 2, let the transmission of the low-pass filter be:

$$F(s) = \frac{1 + t_2s}{1 + t_1s} \quad (1)$$

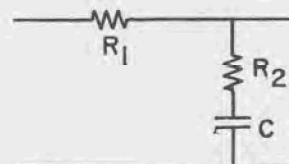


Fig. 5—Lag-network.

This corresponds to the use of a lag network shown in Fig. 5, where

$$t_1 = (R_1 + R_2)C = \text{the time constant of the lag break}$$

$$t_2 = R_2C = \text{the time constant of the lead break.}$$

The transfer function of the system can be calculated to be:

$$\frac{\omega_2(s)}{\omega_1(s)} = \frac{K(1 + t_2s)}{t_1[s^2 + s(1/t_1 + Kt_2/t_1) + (K/t_1)]} \quad (2)$$

where $K = K_1K_2$.

The denominator is of the general form

$$s^2 + 2\zeta\omega_n s + \omega_n^2,$$

where

$$\omega_n = \text{the natural resonant frequency} = \sqrt{\frac{K}{t_1}} \quad (3)$$

$$\zeta = \text{the damping ratio} = \frac{1 + Kt_2}{2\omega_n t_1} \quad (4)$$

¹⁰ J. K. Clapp and F. D. Lewis, "A unique standard-frequency multiplier," 1957 IRE NATIONAL CONVENTION RECORD, pt. 5, pp. 131-136.

¹¹ F. D. Lewis, "New standard frequency multipliers," *General Radio Experimenter*, vol. 32, pp. 3-9; July, 1958.

It will be shown later that ω_n equals ω_c , the open-loop crossover frequency.

Noise Bandwidth: The response of the system to random disturbances on the input can be expressed in terms of its noise bandwidth:^{1,2,5}

$$F_n = \int_0^\infty \left| \frac{\omega_2}{\omega_1} (j\omega) \right|^2 d\omega. \quad (5)$$

This is the area under the square of the closed-loop frequency response curve.

Eq. (2) can be substituted in (5) and the integration carried out to yield^{1,2,5}

$$F_n = \frac{\pi\omega_n}{4\zeta} \left[1 + \left(2\zeta - \frac{\omega_n}{K} \right)^2 \right], \quad (6)$$

or, in terms of $\alpha = t_1/t_2 =$ the network ratio,

$$F_n = \frac{\pi}{2} K \frac{\alpha^2 + Kt_1}{\alpha^2 + \alpha Kt_1}. \quad (7)$$

For the minimum noise bandwidth, it can be shown that the following conditions should be satisfied:²

$$\zeta_0 = \frac{1}{2} \sqrt{1 + \left(\frac{\omega_n}{K} \right)^2}, \quad (8)$$

or, in terms of α ,

$$\alpha_0 = 1 + \sqrt{1 + Kt_1} = 1 + \sqrt{1 + \left(\frac{K}{\omega_n} \right)^2}. \quad (9)$$

These equations can be manipulated to yield the following relationship between the several frequencies of interest:

$$\left(\frac{1}{t_2\omega_n} \right)^2 = 1 + \frac{2}{Kt_1} [1 + \sqrt{1 + Kt_1}]. \quad (10)$$

The left side of this equation is the square of the ratio of the network lead frequency to the resonant frequency. According to the right side, this ratio can never be less than one. This indicates that, in a system with minimum noise bandwidth and for the usual case $K > (1/t_1)$, the resonant frequency $\omega_n = \sqrt{K/t_1}$ equals ω_c , the open-loop crossover frequency, and the crossover frequency occurs in the 12-db-per-octave region of the open-loop gain.

Limiting Cases: With no low-pass filter at all, the noise bandwidth becomes:

$$F_n = \frac{\pi}{2} K \quad (11)$$

This can be seen from (5) by letting $\alpha = 1$. Similarly, with a single-section RC filter

$$F_n = \frac{\pi}{2} K. \quad (12)$$

This can be seen by letting α approach infinity in (5).

This last result is interesting since it shows that as the crossover frequency, $\omega_c = \sqrt{K/t_1}$, is lowered by increasing t_1 , the damping ratio decreases so as to maintain the noise bandwidth constant at $\pi/2K$. In other words, as the crossover frequency is lowered, the peaking of the frequency response curve is increased, maintaining the area under the square of this curve constant.

With the use of the optimized lag network mentioned, the noise bandwidth varies from $\pi/2\omega_c$ for $\omega_c = K$ to $\pi\omega_c$ for $\omega_c \ll K$. The optimum value of ζ varies from 0.707 for $\omega_c = K$ to 0.5 for $\omega_c \ll K$. Similarly, the optimum value of α ranges from 2.4 for $\omega_c = K$, to K/ω_c for $\omega_c \ll K$. This indicates that for values of ω_c much smaller than K the lead break occurs immediately after the crossover frequency. The values are repeated in the following table:

$\frac{\omega_c}{K}$	1	$\ll 1$
ζ_0	0.7	0.5
α_0	2.4	$\frac{K}{\omega_c}$
F_n	$\frac{\pi}{2} \omega_c$	$\pi \omega_c$

Capture Range: Gruen⁵ has shown that the capture range is

$$|\Delta\omega| \simeq \sqrt{2\zeta\omega_n K}, \quad \text{for } \omega_n \ll K \quad (13)$$

or, expressed as a "capture ratio,"

$$\left| \frac{\Delta\omega}{K} \right| = \frac{\text{capture range}}{\text{lock range}} \simeq \sqrt{2\zeta \frac{\omega_n}{K}}. \quad (14)$$

Substituting $\zeta_0 = 0.5$ for $\omega_n \ll K$, this ratio becomes

$$\frac{\text{capture range}}{\text{lock range}} \simeq \sqrt{\frac{\omega_n}{K}} \quad (15)$$

for a system with an optimized lag network.

For a system with a simple RC filter ($t_2 = 0$),

$$\zeta = \frac{1}{2} \frac{\omega_n}{K}. \quad (16)$$

Substituting (16) in (14) yields

$$\frac{\text{capture range}}{\text{lock range}} \simeq \frac{\omega_n}{K}. \quad (17)$$

Capture Time: Richman⁴ states that the capture time depends on the initial frequency offset and the noise bandwidth in the following manner:

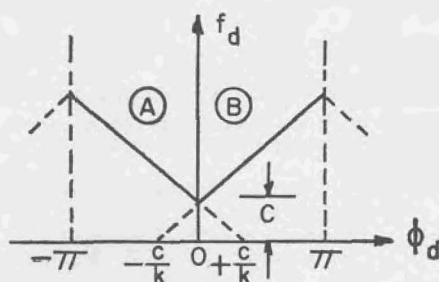


Fig. 6—Composite locking diagram.

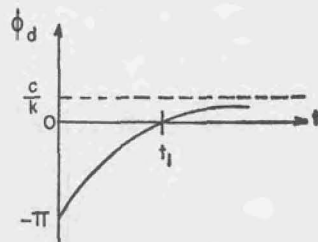


Fig. 7—Phase behavior in Region A.

$$t \simeq 4 \frac{\Delta f^2}{F_n^3}, \quad (18)$$

except near the limit of the capture range, where the capture time approaches infinity.

APPENDIX II

Pulling Behavior of an APC Loop

Fig. 6 illustrates what occurs if the rough tuning of the oscillator is such that the range of the phase-detector-reactance-modulator combination is insufficient to allow locking.

In this case,

$f_d = f - f_0$ = instantaneous frequency difference between oscillator and reference.

$\phi_d = \phi - \phi_0$ = instantaneous phase difference.

c = minimum frequency difference allowed by device.

k = slope of f_d vs ϕ_d characteristic.

Initial Conditions: Assume that at $t=0$, $\phi_d = -\pi$, i.e., the operating point starts at the left extremity of Region A.

In Region A:

$$f_d = -k\phi_d + c,$$

$$f_d = (d/dt)\phi_d.$$

Solving:

$$(d/dt)\phi_d = -k\phi_d + c,$$

$$(d/dt)\phi_d + k\phi_d = c,$$

$$\phi_d = (c/k) + Ae^{-kt}.$$

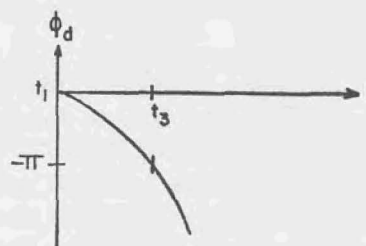


Fig. 8—Phase behavior in Region B.

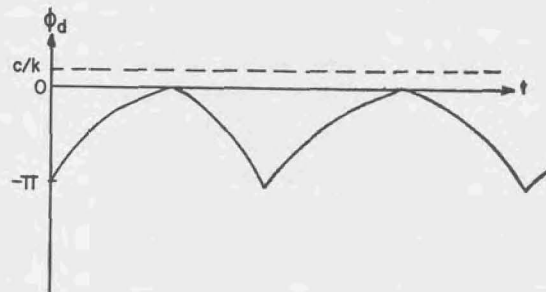


Fig. 9—Steady-state phase behavior.

Substituting initial condition:

$$-\pi = (c/k) + A,$$

$$A = -\pi - (c/k),$$

$$\phi_d = (c/k) - (\pi + c/k)e^{-kt}.$$

The time behavior of ϕ_d is shown in Fig. 7.

However, at $t=t_1$, the operating point enters Region B with initial conditions

$$t = t_1 = t_0'$$

$$\phi_d = 0.$$

In Region B

$$f_d = k\phi_d + c,$$

$$f_d = (d/dt)\phi_d,$$

$$(d/dt)\phi_d = k\phi_d + c,$$

$$(d/dt)\phi_d - k\phi_d = c,$$

$$\phi_d = -(c/k) + Ae^{kt}.$$

Initially,

$$0 = -(c/k) + A,$$

$$A = +(c/k),$$

$$\phi_d = +(c/k)(1 - e^{kt}).$$

The time behavior of ϕ_d is shown in Fig. 8.

However, at t_3 , operating point reaches π , re-entering Region A at the proper initial conditions and the process repeats as shown in Fig. 9.

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Change Notice for Reprint A-79

Harold T. McAleer, author of "A New Look at the Phase-Locked Oscillator," which appeared on pages 1137-1143 of the June, 1959, issue of PROCEEDINGS, has pointed out the following to the *Editor*.

In Appendix I in the section titled "Capture Range" (page 1142), (14) and (17), while correct, are improperly derived. The section should be rewritten to read:

Capture Range: For a system with a simple RC filter ($t_2=0$) and a balanced cosine phase-detector, several authors^{2,6,12} have shown that the capture range $|\Delta\omega|$ is approximately proportional to the crossover frequency ω_n . Expressed as a "capture ratio," the relation becomes:

$$\left| \frac{\Delta\omega}{K} \right| = \frac{\text{capture range}}{\text{lock range}} \simeq A \frac{\omega_n}{K}$$

The value of the multiplying constant A has been variously estimated at values ranging from 0.7 to 1.2.

For a system with an optimized lag network, the capture ratio becomes:^{4,5,12}

$$\frac{\text{capture range}}{\text{lock range}} \simeq B \sqrt{\frac{\omega_n}{K}}$$

For the region $\omega_n/K \ll 1$, the multiplying constant B has been estimated as 1.4.^{4,12}

The value $B=1$ gives a useful approximation for the entire range of values of ω_n/K .

¹² W. E. Ingham, "The Design of an A.P.C. Synchronizing Loop," E.M.I. Res. Labs., Ltd., Hayes, Middlesex, Eng., Rept. No. RW/8; April, 1956.

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